Mark Scheme (Results)

Summer 2019

Pearson Edexcel GCE
In Mathematics (8MA0)
Paper 1 Pure Mathematics

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, a senior examiner must be consulted before a mark is awarded.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to *x* = ...

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to *x* = ...

2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c, \quad q \neq 0$, leading to *x* = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

General Instructions for Marking

1. The total number of marks for the paper is 100

2. These mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations
These are some of the traditional marking abbreviations that will appear in the mark schemes.

- **bod** – benefit of doubt
- **ft** – follow through
- the symbol $\surd$ will be used for correct ft
- **cao** – correct answer only
- **cso** - correct solution only. There must be no errors in this part of the question to obtain this mark
- **isw** – ignore subsequent working
- **awrt** – answers which round to
- **SC**: special case
- **o.e.** – or equivalent (and appropriate)
- **d** or **dep** – dependent
- **indep** – independent
- **dp** decimal places
- **sf** significant figures
- ▯▯The answer is printed on the paper or ag- answer given

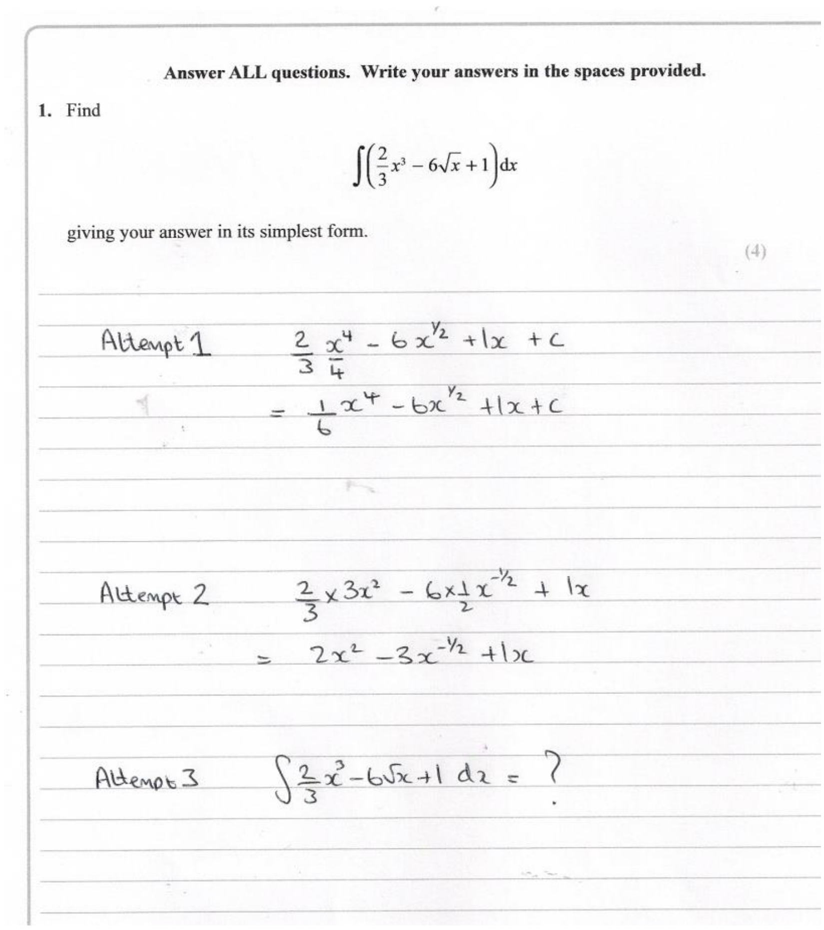
4. All M marks are follow through.

A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but answers that don’t logically make sense e.g. if an answer given for a $\sin\theta$ is >1 or < $-$1, should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.

If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.



Answer ALL questions. Write your answers in the spaces provided.

1. Find

$$\int \left(\frac{2}{3}x^3 - 6\sqrt{x} + 1 \right) dx$$

giving your answer in its simplest form.

(4)

Attempt 1 $\quad \frac{2}{3}\frac{x^4}{4} - 6x^{1/2} + 1x + c$

$= \frac{1}{6}x^4 - 6x^{1/2} + 1x + c$

Attempt 2 $\quad \frac{2}{3} \times 3x^2 - 6 \times \frac{1}{2}x^{-1/2} + 1x$

$= 2x^2 - 3x^{-1/2} + 1x$

Attempt 3 $\quad \int \frac{2}{3}x^3 - 6\sqrt{x} + 1 \, dx = ?$

There are three attempts.

Attempt 1
would score M1 A1 A0 A0

Attempt 2
would score M1 A0 A0 A0

Attempt 3
would score M0 A0 A0 A0

Attempt 3 is not complete. Attempt 2 and 1 are both complete methods. So we score ''Attempt 2'' which is the candidates final solution which is most complete.
Candidate is awarded 1 mark.

7. Ignore wrong working or incorrect statements following a correct answer.

8. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used. If no such alternative answer is provided but the response is deemed to be valid, examiners must escalate the response for a senior examiner to review.

Very brief explanation of the AO's

AO	It is awarded for.....
1.1	Select or carry out routine procedures, recall facts, definitions
2.1	Constructing an argument
2.2	Making a deduction
2.3	Assessing the validity of an argument / Identifying errors in a solution
2.4	Explaining their reasoning
2.5	Using mathematical language/notation correctly
3.1	Translating a problem into maths
3.2	Interpreting solutions or limitations to their problem
3.3	Modelling a problem

3.4	Using a model
3.5	Evaluating the outcome/ refining a model

Question	Scheme	Marks	AOs
1(a)	$2x+4y-3=0 \Rightarrow y = \mp\frac{2}{4}x+...$ Gradient of perpendicular = $\pm\frac{4}{2}$	M1	1.1b
	Either $m=2$ or $y=2x+7$	A1	1.1b
		(2)	
(b)	Combines 'their' $y=2x+7$ with $2x+4y-3=0 \Rightarrow 2x+4(2x+7)-3=0 \Rightarrow x=...$	M1	1.1b
	$x=-2.5$ oe	A1	1.1b
		(2)	
			(4 marks)

Notes

(a)

M1: Attempts to set given equation in the form $y = ax+b$ with $a = \mp\frac{2}{4}$ oe such as $\mp\frac{1}{2}$ **AND** deduces that $m = -\frac{1}{a}$ Condone errors on the "$+b$"

An alternative method is to find both intercepts to get gradient $l_1 = \pm\frac{0.75}{1.5}$ and use the perpendicular gradient rule.

A1: Correct answer. Accept **either** $m=2$ **or** $y=2x+7$

This must be simplified and not left as $m=\frac{4}{2}$ or $m=2x$ unless you see $y=2x+7$.

Watch: There may be candidates who look at $\mathbf{2}x+4y-3=0$ and incorrectly state that the gradient is 2 and use the perpendicular rule to get $m=-\frac{1}{2}$ They will score M0 A0 in (a) and also no marks in (b) as the lines would be parallel. In a case like this don't allow an equation to be "altered"

Candidates who state $m=2$ or $y=2x+7$ **with no incorrect working** can score both marks

(b)

M1: Substitutes their $y=mx+7$ into $2x+4y-3=0$, condoning slips, in an attempt to form and solve an equation in *x*. Alternatively equates their $y=-\frac{1}{2}x+\frac{3}{4}$ with their $y=mx+7$ in an attempt to form and solve, condoning slips, an equation in *x*. Don't be too concerned by the mechanics of the candidates attempt to solve. (E.g. allow solutions from their calculators).

You may see $2x+4y-3=2x-y+7$ with y being found before the value of *x* appears

It cannot be awarded from "unsolvable" equations (e.g. lines that are parallel).

A1: $x=-2.5$

The answer alone can score both marks as long as both equations are correct and no incorrect working is seen.

Remember to isw after the correct answer and ignore any *y* coordinate

Question	Scheme		Marks	AOs
2(i)	$16a^2 = 2\sqrt{a} \Rightarrow a^{\frac{3}{2}} = \frac{1}{8}$	$16a^2 - 2\sqrt{a} = 0$ $\Rightarrow 2a^{\frac{1}{2}}\left(8a^{\frac{3}{2}} - 1\right) = 0$ $\Rightarrow a^{\frac{3}{2}} = \frac{1}{8}$	M1	1.1b
	$\Rightarrow a = \left(\frac{1}{8}\right)^{\frac{2}{3}}$	$\Rightarrow a = \left(\frac{1}{8}\right)^{\frac{2}{3}}$	M1	1.1b
	$\Rightarrow a = \frac{1}{4}$	$\Rightarrow a = \frac{1}{4}$	A1	1.1b
	Deduces that $a = 0$ is a solution		B1	2.2a
			(4)	
(ii)	$b^4 + 7b^2 - 18 = 0 \Rightarrow \left(b^2 + 9\right)\left(b^2 - 2\right) = 0$		M1	1.1b
	$b^2 = -9, 2$		A1	1.1b
	$b^2 = k \Rightarrow b = \sqrt{k}, k > 0$		dM1	2.3
	$b = \sqrt{2}$, $-\sqrt{2}$ only		A1	1.1b
			(4)	
				(8 marks)

Notes

(i)

M1: Combines the two algebraic terms to reach $a^{\pm\frac{3}{2}} = C$ or equivalent such as $\left(\sqrt{a}\right)^3 = C$ $(C \neq 0)$

An alternative is via squaring and combining the algebraic terms to reach $a^{\pm 3} = k, k > 0$

Eg. $...a^4 = ...a \Rightarrow a^{\pm 3} = k$ or $...a^4 = ...a \Rightarrow ...a^4 - ...a = 0 \Rightarrow ...a\left(a^3 - ...\right) = 0 \Rightarrow a^3 = ...$

Allow for slips on coefficients.

M1: Undoes the indices correctly for their $a^{\frac{m}{n}} = C$ (So M0 M1 A0 is possible)
You may even see logs used.

A1: $a = \frac{1}{4}$ and no other solutions apart from 0 Accept exact equivalents Eg 0.25

B1: Deduces that a = 0 is a solution.

(ii)

M1: Attempts to solve as a quadratic equation in b^2
Accept $\left(b^2 + m\right)\left(b^2 + n\right) = 0$ with $mn = \pm 18$ or solutions via the use of the quadratic formula Also allow candidates to substitute in another variable, say $u = b^2$ and solve for u

A1: Correct solution. Allow for $b^2 = 2$ or $u = 2$ with no incorrect solution given.
Candidates can choose to omit the solution $b^2 = -9$ or $u = -9$ and so may not be seen

dM1: Finds at least one solution from their $b^2 = k \Rightarrow b = \sqrt{k}, k > 0$. Allow $b = 1.414$

A1: $b=\sqrt{2}$, $-\sqrt{2}$ only. The solution asks for real values so if $3i$ is given then score A0

Notes on Question 2 continue

Answers with minimal or no working:

In part **(i)**

- no working, just answer(s) with they can score the B1
- If they square and proceed to the quartic equation $256a^4=4a$ oe, and then write down the answers they can have access to all marks.

In part (ii)

- Accept for 4 marks $b^2=2\Rightarrow b=\pm\sqrt{2}$
- No working, no marks.

Question	Scheme	Marks	AOs
3(a)	$x^n \rightarrow x^{n+1}$	M1	1.1b
	$\int\left(\frac{4}{x^3}+kx\right)\mathrm{d}x = -\frac{2}{x^2}+\frac{1}{2}kx^2+c$	A1 A1	1.1b 1.1b
		(3)	
(b)	$\left[-\frac{2}{x^2}+\frac{1}{2}kx^2\right]_{0.5}^{2} = \left(-\frac{2}{2^2}+\frac{1}{2}k\times 4\right)-\left(-\frac{2}{(0.5)^2}+\frac{1}{2}k\times(0.5)^2\right)=8$	M1	1.1b
	$7.5+\frac{15}{8}k=8 \Rightarrow k=...$	dM1	1.1b
	$k=\frac{4}{15}$ oe	A1	1.1b
		(3)	
			(6 marks)

Notes

Mark parts (a) and (b) as one

(a)

M1: For $x^n \rightarrow x^{n+1}$ for either x^{-3} or x^1. This can be implied by the sight of either x^{-2} or x^2.
Condone "unprocessed" values here. Eg. x^{-3+1} and x^{1+1}

A1: Either term correct (un simplified).
Accept $4\times\frac{x^{-2}}{-2}$ or $k\frac{x^2}{2}$ **with** the indices processed.

A1: Correct (and simplified) with $+c$.
Ignore spurious notation e.g. answer appearing with an $\int$ sign or with $\mathrm{d}x$ on the end.
Accept $-\frac{2}{x^2}+\frac{1}{2}kx^2+c$ or exact simplified equivalent such as $-2x^{-2}+k\frac{x^2}{2}+c$

(b)

M1: For substituting both limits into their $-\frac{2}{x^2}+\frac{1}{2}kx^2$, subtracting either way around and setting equal to 8. Allow this when using a changed function. (so the M in part (a) may not have been awarded). Condone missing brackets. Take care here as substituting 2 into the original function gives the same result as the integrated function so you will have to consider both limits.

dM1: For solving a **linear** equation in k. It is dependent upon the previous M only
Don't be too concerned by the mechanics here. Allow for a linear equation in k leading to $k =$

A1: $k=\frac{4}{15}$ or exact equivalent. Allow for $\frac{m}{n}$ where m and n are integers and $\frac{m}{n}=\frac{4}{15}$
Condone the recurring decimal $0.2\dot{6}$ but not 0.266 or 0.267
Please remember to isw after a correct answer

Question	Scheme	Marks	AOs
4 (a)	Attempts $H = mt + c$ with both $(3, 2.35)$ and $(6, 3.28)$	M1	3.3
	Method to find both *m* and *c*	dM1	3.1a
	$H = 0.31t + 1.42$ oe	A1	1.1b
		(3)	
(b)	Uses the model and states that the initial height is their '*b*'	B1ft	3.4
	Compares 140 cm with their 1.42 (m) and makes a valid comment. In the case where $H = 0.31t + 1.42$ it should be this fact supports the use of the linear model as the values are close.	B1ft	3.5a
		(2)	
			(5 marks)

Notes

Mark parts (a) and (b) as one

(a)

M1: For creating a linear model with both pieces of information given.
Eg. Accept sight of $2.35 = 3m + c$ and $3.28 = 6m + c$ Condone slips on the 2.35 and 3.28.
Allow for an attempt at the "gradient" $m = \frac{3.28 - 2.35}{6 - 3} (= 0.31)$ or the intercept.
Allow for a pair of simultaneous in any variable even *x* and *y*

dM1: A full method to find both constants. For simultaneous equations award if they arrive at values for *m* and *c*.
If they attempted the gradient it would be for attempting to find "*c*" using $y = mx + c$ with their *m* and one of the points $(3, 2.35)$ or $(6, 3.28)$

A1: A correct model using allowable/correct variables. $H = 0.31t + 1.42$ Condone $h \leftrightarrow H, t \leftrightarrow T$

Allow equivalents such as $H = \frac{31}{100}t + \frac{142}{100}$, $t = \frac{H - 1.42}{0.31}$ but not $H = \frac{0.93}{3}t + 1.42$

Do not allow $H = 0.31t + 1.42$ m (with the units)

(b) To score any marks in (b) the model must be of the form $H = mt + b$ where $m > 0, b > 0$

B1ft: States or implies that 1.42 (with or without units) or 142 cm (including the units) is the original height or the height when $t = 0$
You should allow statements such as *c* = 1.42 or original height = 1.42 (m)
Follow through on their value of '*c*', so for $H = 0.25t + 1.60$ it is scored for stating the initial height is 1.60 (m) or 160 cm. Do not follow through if $c \leqslant 0$

B1ft: Compares 140 cm with their 1.42 (m) **and** makes a valid comment.
In the case where $H = 0.31t + 1.42$ it should be this fact supports the use of the linear model as the values are close or approximately the same. Allow $1.42\text{m} \approx 1.4\text{m}$ or similar
In the case of $H = 0.25t + 1.60$ it would be for stating that the fact that it does not support the use of the model as the values are too different. If they state $1.60 > 1.40$ this is insufficient. They cannot just state that they are not the same. It must be implied that there is a significant difference.
As a rule of thumb use "good model" for between 135cm and 145 cm.

This requires a correct calculation for their *H*, a correct statement with an appreciation shown for the units and a correct conclusion.
Notes on Question 4 continue
SC B1 B0 Award SC for incomplete answers which suggest the candidate knows what to do. Eg. In (b) $H = 0.31t + 1.42$ followed by in (c) It supports the model as when $t = 0$ it is approximately 140 cm

Question	Scheme	Marks	AOs
5(a)	$x^n \to x^{n-1}$	M1	1.1b
	$\left(\frac{dy}{dx}\right) = 6x - \frac{24}{x^2}$	A1 A1	1.1b 1.1b
		(3)	
(b)	Attempts $6x - \frac{24}{x^2} > 0 \Rightarrow x >$	M1	1.1b
	$x > \sqrt[3]{4}$ or $x \geqslant \sqrt[3]{4}$	A1	2.5
		(2)	
			(5 marks)

Notes

(a)

M1: $x^n \to x^{n-1}$ for any correct index of x. Score for $x^2 \to x$ or $x^{-1} \to x^{-2}$

Allow for unprocessed indices. $x^2 \to x^{2-1}$ oe

A1: Sight of either $6x$ or $-\frac{24}{x^2}$ which may be un simplified.

Condone an additional term e.g. + 2 for this mark

The indices now must have been processed

A1: $\frac{dy}{dx} = 6x - \frac{24}{x^2}$ or exact simplified equivalent. Eg accept $\frac{dy}{dx} = 6x^1 - 24x^{-2}$

You do not need to see the $\frac{dy}{dx}$ and you should isw after a correct simplified answer.

(b)

M1: Sets an allowable $\frac{dy}{dx}$**...**0 and proceeds to x... via an allowable intermediate equation

$\frac{dy}{dx}$ must be in the form $Ax + Bx^{-2}$ where $A, B \neq 0$

and the intermediate equation must be of the form $x^p ... q$ oe

Do not be concerned by either the processing, an equality or a different inequality.

It may be implied by $x =$ awrt 1.59

A1: $x > \sqrt[3]{4}$ or $x \geqslant \sqrt[3]{4}$ oe such as $x > 4^{\frac{1}{3}}$ or $x \geqslant 2^{\frac{2}{3}}$

Question	Scheme	Marks	AOs
6 (a)	Uses $18\sqrt{3}=\frac{1}{2}\times 2x\times 3x\times\sin 60°$	M1	1.1a
	Sight of $\sin 60°=\frac{\sqrt{3}}{2}$ and proceeds to $x^2=k$ oe	M1	1.1b
	$x=\sqrt{12}=2\sqrt{3}$ *	A1*	2.1
		(3)	
(b)	Uses $BC^2=\left(6\sqrt{3}\right)^2+\left(4\sqrt{3}\right)^2-2\times 6\sqrt{3}\times 4\sqrt{3}\times\cos 60°$	M1	1.1b
	$BC^2=84$	A1	1.1b
	$BC=2\sqrt{21}$ (cm)	A1	1.1b
		(3)	
			(6 marks)

Notes

(a)

M1: Attempts to use the formula $A=\frac{1}{2}ab\sin C$.

If the candidate writes $18\sqrt{3}=\frac{1}{2}\times 5x\times\sin 60°$ **without** sight of a previous correct line then this would be M0

M1: Sight of $\sin 60°=\frac{\sqrt{3}}{2}$ or awrt 0.866 and proceeds to $x^2=k$ oe such as $px^2=q$

This may be awarded from the correct formula or $A=ab\sin C$

A1*: Look for $x^2=12\Rightarrow x=2\sqrt{3}$, $x^2=4\times 3\Rightarrow x=2\sqrt{3}$ or $x=\sqrt{12}=2\sqrt{3}$

This is a given answer and all aspects must be correct including one of the above intermediate lines. It cannot be scored by using decimal equivalents to $\sqrt{3}$

Alternative using the given answer of $x=2\sqrt{3}$

M1: Attempts to use the formula $A=\frac{1}{2}\times 4\sqrt{3}\times 6\sqrt{3}\sin 60°$ oe

M1: Sight of $\sin 60°=\frac{\sqrt{3}}{2}$ and proceeds to $A=18\sqrt{3}$

A1*: Concludes that $x=2\sqrt{3}$

(b)

M1: Attempts the cosine rule with the sides in the correct position.

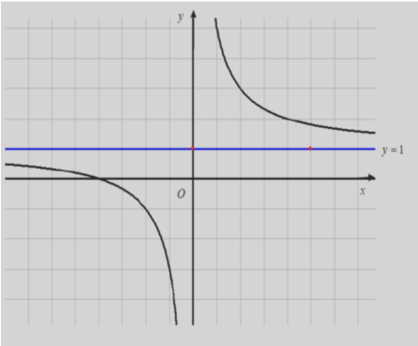
This can be scored from $BC^2=(3x)^2+(2x)^2-2\times 3x\times 2x\times\cos 60°$ as long as there is some attempt to substitute x in later. Condone slips on the squaring

A1: $BC^2=84$ Accept $BC^2=7\times 12$, $BC=\sqrt{84}$ or $BC=2\sqrt{21}$

If they replace the surds with decimals they can score the A1 for $BC^2=$ awrt 84.0

A1: $BC=2\sqrt{21}$

Condone other variables, say $x=2\sqrt{21}$, but it cannot be scored via decimals.

Question	Scheme	Marks	AOs
7 (a)	 $\frac{1}{x}$ shape in 1st quadrant	M1	1.1b
	Correct	A1	1.1b
	Asymptote $y=1$	B1	1.2
		(3)	
(b)	Combines equations $\Rightarrow \frac{k^2}{x}+1=-2x+5$	M1	1.1b
	$(\times x)\Rightarrow k^2+1x=-2x^2+5x \Rightarrow 2x^2-4x+k^2=0$ *	A1*	2.1
		(2)	
(c)	Attempts to set $b^2-4ac=0$	M1	3.1a
	$8k^2=16$	A1	1.1b
	$k=\pm\sqrt{2}$	A1	1.1b
		(3)	
			(8 marks)

Notes

(a)

M1: For the shape of a $\frac{1}{x}$ type curve in Quadrant 1. It must not cross either axis and have acceptable curvature. Look for a negative gradient changing from $-\infty$ to 0 condoning "slips of the pencil". (See Practice and Qualification for clarification)

A1: Correct shape and position for both branches.
It must lie in Quadrants 1, 2 and 3 and have the correct curvature including asymptotic behaviour

B1: Asymptote given as $y=1$. This could appear on the diagram or within the text.
Note that the curve does not need to be asymptotic at $y=1$ but this must be the only horizontal asymptote offered by the candidate.

(b)

M1: Attempts to combine $y=\frac{k^2}{x}+1$ with $y=-2x+5$ to form an equation in just x

A1*: Multiplies by x (the processed line must be seen) and proceeds to given answer with no slips.
Condone if the order of the terms are different $2x^2+k^2-4x=0$

(c)

M1: Deduces that $b^2-4ac=0$ or equivalent for **the given equation.**
If *a*, *b* and *c* are stated only accept $a=2, b=\pm4, c=k^2$ so $4^2-4\times2\times k^2=0$
Alternatively completes the square $x^2-2x+\frac{1}{2}k^2=0 \Rightarrow (x-1)^2=1-\frac{1}{2}k^2 \Rightarrow$ "$1-\frac{1}{2}k^2$" $=0$

A1: $8k^2=16$ or exact simplified equivalent. Eg $8k^2-16=0$

If *a*, *b* and *c* are stated they must be correct. Note that b^2 appearing as 4^2 is correct
Note on Question 7 continue
A1: $k = \pm\sqrt{2}$ and following correct *a*, *b* and *c* if stated

A solution via differentiation would be awarded as follows

M1: Sets the gradient of the curve $= -2 \Rightarrow -\frac{k^2}{x^2} = -2 \Rightarrow x = (\pm)\frac{k}{\sqrt{2}}$ oe and attempts to substitute into $2x^2 - 4x + k^2 = 0$

A1: $2k^2 = (\pm)2\sqrt{2}k$ oe

A1: $k = \pm\sqrt{2}$

Question	Scheme	Marks	AOs
8(a)	2^6 or 64 as the constant term	B1	1.1b
	$\left(2+\frac{3x}{4}\right)^6 = ...+{}^6C_1 2^5\left(\frac{3x}{4}\right)^1+{}^6C_2 2^4\left(\frac{3x}{4}\right)^2+...$ $= ...+6\times 2^5\left(\frac{3x}{4}\right)^1+\frac{6\times 5}{2}\times 2^4\left(\frac{3x}{4}\right)^2+...$	M1 A1	1.1b 1.1b
	$= 64+144x+135x^2+...$	A1	1.1b
		(4)	
(b)	$\frac{3x}{4}=-0.075 \Rightarrow x=-0.1$ So find the value of $64+144x+135x^2$ with $x=-0.1$	B1ft	2.4
		(1)	
			(5 marks)

Notes

(a)

B1: Sight of either 2^6 or 64 as the constant term

M1: An attempt at the binomial expansion. This may be awarded for a correct attempt at either the second **OR** third term. Score for the correct binomial coefficient with the correct power of 2 and the correct power of $\frac{3x}{4}$ condoning slips. Correct bracketing is not essential for this M mark.

Condone ${}^6C_2 2^4 \frac{3x}{4}^2$ for this mark

A1: Correct (unsimplified) second **AND** third terms.

The binomial coefficients must be processed to numbers /numerical expression e.g $\frac{6!}{4!2!}$ or $\frac{6\times 5}{2}$

They cannot be left in the form ${}^6C_1$ and/or $\binom{6}{2}$

A1: $64+144x+135x^2+...$ Ignore any terms after this. Allow to be written $64, 144x, 135x^2$

(b)

B1ft: $x=-0.1$ or $-\frac{1}{10}$ **with** a comment about substituting this into their $64+144x+135x^2$

If they have written (a) as $64, 144x, 135x^2$ candidate would need to say substitute $x=-0.1$ into the sum of the first three terms.

As they do not have to perform the calculation allow

Set $2+\frac{3x}{4}=1.925$, solve for *x* and then substitute this value into the expression from (a)

If a value of *x* is found then it must be correct

Alternative to part (a)

$$\left(2+\frac{3x}{4}\right)^6 = 2^6\left(1+\frac{3x}{8}\right)^6 = 2^6\left(1+{}^6C_1\left(\frac{3x}{8}\right)^1+{}^6C_2\left(\frac{3x}{8}\right)^2+...\right)$$

B1: Sight of either 2^6 or 64

M1: An attempt at the binomial expansion. This may be awarded for either the second or third term. Score for the correct binomial coefficient with the correct power of $\frac{3x}{8}$ Correct bracketing is not essential for this mark.
A1: A correct attempt at the binomial expansion on the second and third terms.
A1: $64+144x+135x^2+...$ Ignore any terms after this.

Question	Scheme	Marks	AOs
9 (a)	117 tonnes	B1	3.4
		(1)	
(b)	1200 tonnes	B1	2.2a
		(1)	
(c)	Attempts $\{1200-3\times(5-20)^2\}-\{1200-3\times(4-20)^2\}$	M1	3.1a
	93 tonnes	A1	1.1b
		(2)	
(d)	States the model is only valid for values of *n* such that $n \leqslant 20$	B1	3.5b
	States that the total amount mined cannot decrease	B1	2.3
		(2)	
			(6 marks)

Notes

Note: Only withhold the mark for a lack of tonnes, once, the first time that it occurs.

(a)
B1: 117 tonnes or 117 t.
(b)
B1: 1200 tonnes or 1200 t.
(c)
M1: Attempts $T_5 - T_4 = \{1200-3\times(5-20)^2\}-\{1200-3\times(4-20)^2\}$ May be implied by $525-432$

Condone for this mark an attempt at $T_4 - T_3 = \{1200-3\times(4-20)^2\}-\{1200-3\times(3-20)^2\}$

A1: 93 tonnes or 93 t
(d)
For one mark

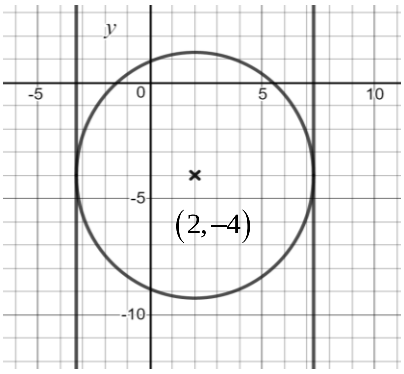
Shows an appreciation of the model

- States $n \leqslant 20$ or $n < 20$
- Condone for one mark $n \leqslant 40$ or $n < 40$ **with** "the mass of tin mined cannot be negative" oe
- Condone for one mark $n = 40$ **with** a statement that "the mass of tin mined becomes 0" oe
- after 20 years the (total) amount of tin mined starts to go down (*n* may not be mentioned and total may be missing)
- after 20 years the (total) mass reaches a maximum value. (Similar to above)
- States T_{max} is reached when $n = 20$

For two marks

States the limitation on *n* and explains fully. (Total mass, not mass must be used)

- States that $n \leqslant 20$ and explains that the total mass of tin cannot decrease.
- Alternatively states that *n* cannot be more than 20 and the total mass of tin would be decreasing
- $0 < n \leqslant 20$ as the maximum total amount of tin mined is reached at 20 years

Question	Scheme	Marks	AOs
10(a)	$x^2+y^2-4x+8y-8=0$		
	Attempts $(x-2)^2+(y+4)^2-4-16-8=0$	M1	1.1b
	(i) Centre $(2,-4)$	A1	1.1b
	(ii) Radius $\sqrt{28}$ oe Eg $2\sqrt{7}$	A1	1.1b
		(3)	
(b)	 Attempts to add/subtract '*r*' from '2'	M1	3.1a
	$k=2\pm\sqrt{28}$	A1ft	1.1b
		(2)	
			(5 marks)

Notes

(a)

M1: Attempts to complete the square. Look for $(x\pm2)^2+(y\pm4)^2$ ...

If a candidate attempts to use $x^2+y^2+2gx+2fy+c=0$ then it may be awarded for a centre of $(\pm2,\pm4)$ Condone $a=\pm2, b=\pm4$

A1: Centre $(2,-4)$ This may be written separately as $x=2, y=-4$ BUT $a=2, b=-4$ is A0

A1: Radius $\sqrt{28}$ or $2\sqrt{7}$ isw after a correct answer

(b)

M1: Attempts to add **or** subtract their radius from their 2.

Alternatively substitutes $y=-4$ into circle equation and finds *x*/*k* by solving the quadratic equation by a suitable method.

A third (and more difficult) method would be to substitute $x=k$ into the equation to form a quadratic eqn in $y \Rightarrow y^2+8y+k^2-4k-8=0$ and use the fact that this would have one root.

E.g. $b^2-4ac=0 \Rightarrow 64-4(k^2-4k-8)=0 \Rightarrow k=..$ Condone slips but the method must be sound.

A1ft: $k=2+\sqrt{28}$ and $k=2-\sqrt{28}$ Follow through on their 2 and their $\sqrt{28}$

If decimals are used the values must be calculated. Eg $k=2\pm5.29 \rightarrow k=7.29, k=-3.29$

Accept just $2\pm\sqrt{28}$ or equivalent such as $2\pm2\sqrt{7}$

Condone $x=2+\sqrt{28}$ and $x=2-\sqrt{28}$ but not $y=2+\sqrt{28}$ and $y=2-\sqrt{28}$

Question	Scheme	Marks	AOs
11 (a)	Attempts $f(4) = 2\times4^3 - 13\times4^2 + 8\times4 + 48$	M1	1.1b
	$f(4) = 0 \Rightarrow (x-4)$ is a factor	A1	1.1b
		(2)	
(b)	$2x^3 - 13x^2 + 8x + 48 = (x-4)(2x^2 ... x - 12)$	M1	2.1
	$= (x-4)(2x^2 - 5x - 12)$	A1	1.1b
	Attempts to factorise quadratic factor or solve quadratic eqn	dM1	1.1b
	$f(x) = (x-4)^2(2x+3) \Rightarrow f(x) = 0$ has only two roots, 4 and -1.5	A1	2.4
		(4)	
(c)	Deduces either three roots or deduces that $f(x)$ is moved down two units	M1	2.2a
	States three roots, as when $f(x)$ is moved down two units there will be three points of intersection (with the x - axis)	A1	2.4
		(2)	
(d)	For sight of $k = \pm4, \pm\frac{3}{2}$	M1	1.1b
	$k = 4, -\frac{3}{2}$	A1ft	1.1b
		(2)	
			(10 marks)

Notes

(a)

M1: Attempts to calculate $f(4)$.

Do not accept $f(4) = 0$ without sight of embedded values or calculations.

If values are not embedded look for two correct terms from $f(4) = 128 - 208 + 32 + 48$

Alternatively attempts to divide by $(x-4)$. Accept via long division or inspection.

See below for awarding these marks.

A1: Correct reason with conclusion. Accept $f(4) = 0$, hence factor as long as M1 has been scored.

This should really be stated on one line after having performed a correct calculation. It could appear as a preamble if the candidate states "If $f(4) = 0$, then $(x-4)$ is a factor before doing the calculation and then writing hence proven or ✓ oe.

If division/inspection is attempted it must be correct and there must be some attempt to explain why they have shown that $(x-4)$ is a factor. Eg Via division they must state that there is no remainder, hence factor

(b)

M1: Attempts to find the quadratic factor by inspection (correct first and last terms) or by division (correct first two terms)

Notes on Question 11 continue

So for inspection award for $2x^3 - 13x^2 + 8x + 48 = (x-4)(2x^2 ... x \pm 12)$

$$\begin{array}{r} 2x^2 - 5x \\ x-4 \overline{)\,2x^3 - 13x^2 + 8x + 48} \end{array}$$

For division look for $\quad \begin{array}{r} 2x^3 - 8x^2 \\ \hline -5x^2 \end{array}$

A1: Correct quadratic factor $(2x^2 - 5x - 12)$ For division award for sight of this "in the correct place" You don't have to see it paired with the $(x-4)$ for this mark.

If a student has used division in part (a) they can score the M1 A1 in (b) as soon as they start attempting to factorise their $(2x^2 - 5x - 12)$.

dM1: Correct attempt to solve or factorise their $(2x^2 - 5x - 12)$ including use of formula

Apply the usual rules $(2x^2 - 5x - 12) = (ax+b)(cx+d)$ where $ac = \pm 2$ and $bd = \pm 12$

Allow the candidate to move from $(x-4)(2x^2 - 5x - 12)$ to $(x-4)^2(2x+3)$ for this mark.

A1: Via factorisation

Factorises twice to $f(x) = (x-4)(2x+3)(x-4)$ or $f(x) = (x-4)^2(2x+3)$ or

$f(x) = 2(x-4)^2\left(x + \frac{3}{2}\right)$ followed by a valid explanation why there are only two roots.

The explanation can be as simple as

- hence $x = 4$ and $-\frac{3}{2}$ (only). The roots must be correct
- only two distinct roots as 4 is a repeated root

There must be some understanding between roots and factors.

E.g. $\quad f(x) = (x-4)^2(2x+3)$

only two distinct roots is insufficient.

This would require two distinct factors, so there are two distinct roots.

Via solving.

Factorsises to $(x-4)(2x^2 - 5x - 12)$ and solves $2x^2 - 5x - 12 = 0 \Rightarrow x = 4, -\frac{3}{2}$ followed by an explanation that the roots are $4, 4, -\frac{3}{2}$ so only two distinct roots.

Note that this question asks the candidate to use algebra so you cannot accept any attempt to use their calculators to produce the answers.

(c)

M1: For a valid **deduction.**

Accept **either** there are 3 roots **or** states that it is a solution of $f(x) = 2$ or $f(x) - 2 = 0$

A1: Fully explains:

Eg. States three roots, as $f(x)$ is moved down by **two** units (giving three points of intersection with the x - axis)

Eg. States three roots, as it is where $f(x) = 2$ (You may see $y = 2$ drawn on the diagram)

Notes on Question 11 continue
(d) **M1:** For sight of ± 4 **and** $\pm\frac{3}{2}$ Follow through on ± their roots. **A1ft:** $k = 4, -\frac{3}{2}$ Follow through on their roots. Accept $4, -\frac{3}{2}$ but not $x = 4, -\frac{3}{2}$

Question	Scheme	Marks	AOs
12(a)	$\dfrac{10\sin^2\theta-7\cos\theta+2}{3+2\cos\theta}\equiv\dfrac{10\left(1-\cos^2\theta\right)-7\cos\theta+2}{3+2\cos\theta}$	M1	1.1b
	$\equiv\dfrac{12-7\cos\theta-10\cos^2\theta}{3+2\cos\theta}$	A1	1.1b
	$\equiv\dfrac{(3+2\cos\theta)(4-5\cos\theta)}{3+2\cos\theta}$	M1	1.1b
	$\equiv 4-5\cos\theta$ *	A1*	2.1
		(4)	
(b)	$4+3\sin x=4-5\cos x\Rightarrow\tan x=-\dfrac{5}{3}$	M1	2.1
	$x=\text{awrt }121°,301°$	A1 A1	1.1b 1.1b
		(3)	
			(7 marks)

Notes

(a)

M1: Uses the identity $\sin^2\theta=1-\cos^2\theta$ within the fraction

A1: Correct (simplified) expression in just $\cos\theta$ $\dfrac{12-7\cos\theta-10\cos^2\theta}{3+2\cos\theta}$ or exact equivalent such as $\dfrac{(3+2\cos\theta)(4-5\cos\theta)}{3+2\cos\theta}$ Allow for $\dfrac{12-7u-10u^2}{3+2u}$ where they introduce $u=\cos\theta$

We would condone mixed variables here.

M1: A correct attempt to factorise the numerator, usual rules. Allow candidates to use $u=\cos\theta$ oe

A1*: A fully correct proof with correct notation and no errors.

Only withhold the last mark for (1) Mixed variable e.g. θ and x's (2) Poor notation $\cos\theta^2\leftrightarrow\cos^2\theta$ or $\sin^2=1-\cos^2$ within the solution.

Don't penalise incomplete lines if it is obvious that it is just part of their working

E.g. $\dfrac{10\sin^2\theta-7\cos\theta+2}{3+2\cos\theta}\equiv\dfrac{10\left(1-\cos^2\theta\right)-7\cos\theta+2}{}=\dfrac{12-7\cos\theta-10\cos^2\theta}{3+2\cos\theta}$

(b)

M1: Attempts to use part (a) and proceeds to an equation of the form $\tan x=k,\quad k\neq 0$

Condone $\theta\leftrightarrow x$ Do not condone $a\tan x=0\Rightarrow\tan x=b\Rightarrow x=...$

Alternatively squares $3\sin x=-5\cos x$ and uses $\sin^2 x=1-\cos^2 x$ oe to reach $\sin x=A,-1<A<1$ or $\cos x=B,-1<B<1$

A1: Either $x=\text{awrt }121°$ **or** $301°$. Condone awrt 2.11 or 5.25 which are the radian solutions

A1: Both $x=\text{awrt }121°$ **and** $301°$ and no other solutions.

Answers without working, or with no incorrect working in (b).

Question states hence or otherwise so allow

For 3 marks both $x=\text{awrt }121°$ **and** $301°$ and no other solutions.

For 1 marks scored SC 100 for either $x=\text{awrt }121°$ **or** $301°$

Notes on Question 12 continue
Alternative proof in part (a): **M1:** Multiplies across and form 3TQ in $\cos\theta$ on rhs $10\sin^2\theta - 7\cos\theta + 2 = (4-5\cos\theta)(3+2\cos\theta) \Rightarrow 10\sin^2\theta - 7\cos\theta + 2 = A\cos^2\theta + B\cos\theta + C$ **A1:** Correct identity formed $10\sin^2\theta - 7\cos\theta + 2 = -10\cos^2\theta - 7\cos\theta + 12$ **dM1:** Uses $\cos^2\theta = 1-\sin^2\theta$ on the rhs or $\sin^2\theta = 1-\cos^2\theta$ on the lhs Alternatively proceeds to $10\sin^2\theta + 10\cos^2\theta = 10$ and makes a statement about $\sin^2\theta + \cos^2\theta = 1$ oe **A1*:** Shows that $(4-5\cos\theta)(3+2\cos\theta) \equiv 10\sin^2\theta - 7\cos\theta + 2$ oe AND makes a minimal statement "hence true"

	Scheme	Marks	AOs
13.	The overall method of finding the x coordinate of A.	M1	3.1a
	$y = 2x^3 - 17x^2 + 40x \Rightarrow \frac{dy}{dx} = 6x^2 - 34x + 40$	B1	1.1b
	$\frac{dy}{dx} = 0 \Rightarrow 6x^2 - 34x + 40 = 0 \Rightarrow 2(3x-5)(x-4) = 0 \Rightarrow x = ...$	M1	1.1b
	Chooses $x = 4$ ~~$x = \frac{5}{3}$~~	A1	3.2a
	$\int 2x^3 - 17x^2 + 40x\,dx = \left[\frac{1}{2}x^4 - \frac{17}{3}x^3 + 20x^2\right]$	B1	1.1b
	Area $= \frac{1}{2}(4)^4 - \frac{17}{3}(4)^3 + 20(4)^2$	M1	1.1b
	$= \frac{256}{3}$ *	A1*	2.1
		(7)	
			(7 marks)

Notes

M1: An overall problem -solving method mark to find the minimum point. To score this you need to see

- an attempt to differentiate with at least **two correct terms**
- an attempt to set their $\frac{dy}{dx} = 0$ and then solve to find x. Don't be overly concerned by the mechanics of this solution

B1: $\left(\frac{dy}{dx} =\right) 6x^2 - 34x + 40$ which may be unsimplified

M1: Sets their $\frac{dy}{dx} = 0$, which must be a 3TQ in x, and attempts to solve via factorisation, formula or calculator. If a calculator is used to find the roots, they must be correct for their quadratic.
If $\frac{dy}{dx}$ is correct allow them to just choose the root 4 for M1 A1. Condone $(x-4)\left(x - \frac{5}{3}\right)$

A1: Chooses $x = 4$ This may be awarded from the upper limit in their integral

B1: $\int 2x^3 - 17x^2 + 40x\,dx = \left[\frac{1}{2}x^4 - \frac{17}{3}x^3 + 20x^2\right]$ which may be unsimplified

M1: Correct attempt at area. There may be slips on the integration but expect **two correct terms**
The upper limit used must be their larger solution of $\frac{dy}{dx} = 0$ and the lower limit used must be 0.
So if their roots are 6 and 10, then they must use 10 and 0. If only one value is found then the limits must be 0 to that value.
Expect to see embedded or calculated values.
Don't accept $\int_0^4 2x^3 - 17x^2 + 40x\,dx = \frac{256}{3}$ without seeing the integration and the embedded or calculated values

A1*: Area $= \frac{256}{3}$ **with** correct notation and no errors. Note that this is a given answer.

Notes on Question 13 continue
For correct notation expect to see • $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}}{\mathrm{d}x}$ used correctly at least once. If $\mathrm{f}(x)$ is used accept $\mathrm{f}'(x)$. Condone y' • $\int 2x^3 - 17x^2 + 40x \, \mathrm{d}x$ used correctly at least once with or without the limits.

Question	Scheme	Marks	AOs
14 (a)	(£)18 000	B1	3.4
		(1)	
(b)	(i) $\frac{dV}{dt} = -3925e^{-0.25t}$	M1 A1	3.1b 1.1b
	Sets $-3925e^{-0.25T} = -500 \Rightarrow 3925e^{-0.25T} = 500$ * cso	A1*	3.4
	(ii) $e^{-0.25T} = 0.127... \Rightarrow -0.25T = \ln 0.127...$	M1	1.1b
	$T = 8.24$ (awrt)	A1	1.1b
	8 years 3 months	A1	3.2a
		(6)	
(c)	2 300	B1	1.1b
		(1)	
(d)	Any suitable reason such as • Other factors affect price such as condition/mileage • If the car has had an accident it will be worth less than the model predicts • The price may go up in the long term as it becomes rare • £2300 is too large a value for a car's scrap price. Most cars scrap for around £400	B1	3.5b
		(1)	

(9 marks)

Notes

(a)

B1: £18 000 There is no requirement to have the units

(b)(i)

M1: Award for making the link between gradient and rate of change.

Score for attempting to differentiate V to $\frac{dV}{dt} = ke^{-0.25t}$ An attempt at both sides are required.

For the left hand side you may condone attempts such as $\frac{dy}{dx}$

A1: Achieves $\frac{dV}{dt} = -3925e^{-0.25t}$ or $\frac{dV}{dt} = 15700 \times -0.25e^{-0.25t}$ with both sides correct

A1*: Sets $-3925e^{-0.25T} = -500$ oe and proceeds to $3925e^{-0.25T} = 500$

This is a given answer and to achieve this mark, all aspects must be seen and be correct.

t must be changed to T at some point even if just at the end of their solution/proof

SC: Award SC 110 candidates who simply write

$-3925e^{-0.25T} = -500 \Rightarrow 3925e^{-0.25T} = 500$ without any mention or reference to $\frac{dV}{dt}$

Or $15700 \times -0.25e^{-0.25t} = -500 \Rightarrow 3925e^{-0.25T} = 500$ without any mention or reference to $\frac{dV}{dt}$

(b)(ii)

M1: Proceeds from $e^{-0.25T} = A, A > 0$ using ln's to $\pm 0.25T = ..$

Alternatively takes lns first $3925e^{-0.25T} = 500 \Rightarrow \ln 3925 - 0.25T = \ln 500 \Rightarrow \pm 0.25T = ...$

but $3925e^{-0.25T} = 500 \Rightarrow \ln 3925 \times -0.25T = \ln 500 \Rightarrow \pm 0.25T = ...$ is M0

A1: $T =$ awrt 8.24 or $-\frac{1}{0.25}\ln\left(\frac{20}{157}\right)$ Allow $t =$ awrt 8.24

Notes on Question 14 continue
A1: 8 years 3 months. Correct answer and solution only Answers obtained numerically score 0 marks. The M mark must be scored. **(c)** **B1:** 2 300 but condone £ 2 300 **(d)** **B1:** Any suitable reason. See scheme Accept "Scrappage" schemes may pay more (or less) than £ 2 300. Do not accept "does not take into account inflation" It asks for a limitation of the model so candidates cannot score marks by suggesting other suitable models " the value may fall by the same amount each year"

Question 15

Score as below so M0 A0 M1 A1 or M1 A0 M1 A1 are not possible

Generally the marks are awarded for
M1: Suitable approach to answer the question for *n* being even **OR** odd
A1: Acceptable proof for *n* being even **OR** odd
M1: Suitable approach to answer the question for *n* being even **AND** odd
A1: Acceptable proof for *n* being even **AND** odd **WITH** concluding statement.

There is no merit in a

- student taking values, or multiple values, of *n* and then drawing conclusions. So $n=5 \Rightarrow n^3+2=127$ which is not a multiple of 8 scores no marks.
- student using divided when they mean divisible. Eg. "Odd numbers cannot be divided by 8" is incorrect. We need to see either "odd numbers are not divisible by 8" or "odd numbers cannot be divided by 8 **exactly**"
- stating $\frac{n^3+2}{8}=\frac{1}{8}n^3+\frac{1}{4}$ which is not a whole number
- stating $\frac{(n+1)^3+2}{8}=\frac{1}{8}n^3+\frac{3}{8}n^2+\frac{3}{8}n+\frac{3}{8}$ which is not a whole number

There must be an attempt to generalise either logic or algebra.

Example of a logical approach

Logical approach	States that if *n* is odd, n^3 is odd	M1	2.1
	so n^3+2 is odd and therefore cannot be divisible by 8	A1	2.2a
	States that if *n* is even, n^3 is a multiple of 8	M1	2.1
	so n^3+2 cannot be a multiple of 8 So (Given $n \in \mathbb{N}$), n^3+2 is not divisible by 8	A1	2.2a
		(4)	
			4 marks

First M1: States the result of cubing an odd or an even number
First A1: Followed by the result of adding two and gives a valid reason why it is not divisible by 8.
So for odd numbers accept for example

"odd number + 2 is still odd and odd numbers are not divisible by 8"

"n^3+2 is odd and cannot be divided by 8 **exactly''**

and for even numbers accept

"a multiple of 8 add 2 is not a multiple of 8, so n^3+2 is not divisible by 8"

"if n^3 is a multiple of 8 then n^3+2 cannot be divisible by 8

Second M1: States the result of cubing an odd and an even number
Second A1: Both valid reasons must be given followed by a concluding statement.

Example of algebraic approaches

Question	Scheme	Marks	AOs
15 **Algebraic approach**	(If n is even,) $n=2k$ and $n^3+2=(2k)^3+2=8k^3+2$	M1	2.1
	Eg.'This is 2 more than a multiple of 8, hence not divisible by 8' Or 'as $8k^3$ is divisible by 8 , $8k^3+2$ isn't'	A1	2.2a
	(If n is odd,) $n=2k+1$ and $n^3+2=(2k+1)^3+2$	M1	2.1
	$=8k^3+12k^2+6k+3$ which is an even number add 3, therefore odd. Hence it is not divisible by 8 So (given $n\in\mathbb{N}$,) n^3+2 is not divisible by 8	A1	2.2a
		(4)	
Alt algebraic approach	(If n is even,) $n=2k$ and $\frac{n^3+2}{8}=\frac{(2k)^3+2}{8}=\frac{8k^3+2}{8}$	M1	2.1
	$=k^3+\frac{1}{4}$ oe which is not a whole number and hence not divisible by 8	A1	2.2a
	(If n is odd,) $n=2k+1$ and $\frac{n^3+2}{8}=\frac{(2k+1)^3+2}{8}$	M1	2.1
	$=\frac{8k^3+12k^2+6k+3}{8}$ ** The numerator is odd as $8k^3+12k^2+6k+3$ is an even number +3 hence not divisible by 8 So (Given $n\in\mathbb{N}$,) n^3+2 is not divisible by 8	A1	2.2a
		(4)	

Notes

Correct expressions are required for the M's. There is no need to state "**If *n* is even**," $n=2k$ and "**If *n* is odd,** $n=2k+1$" for the two M's as the expressions encompass all numbers. However the concluding statement must attempt to show that it has been proven for all $n\in\mathbb{N}$

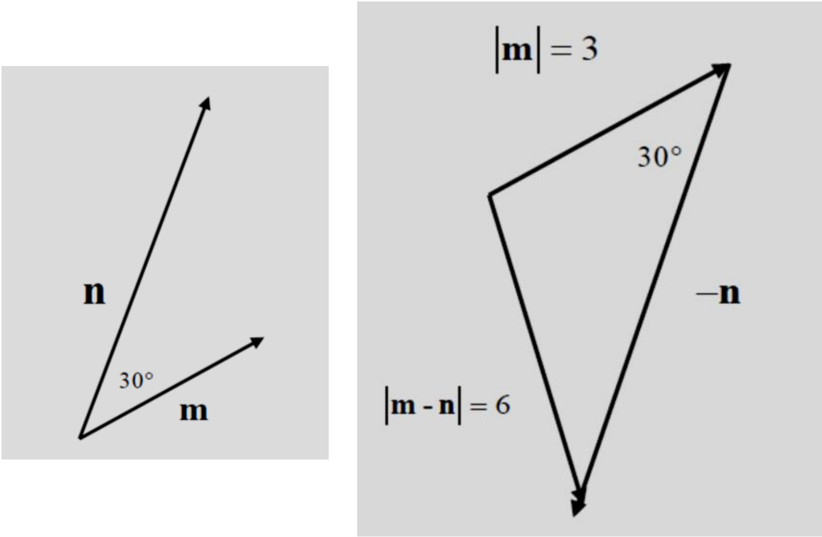
Some students will use $2k-1$ for odd numbers

There is no requirement to change the variable. They may use $2n$ and $2n\pm1$

Reasons must be correct. Don't accept $8k^3+2$ cannot be divided by 8 for example. (It can!)

Also **"$=\frac{8k^3+12k^2+6k+3}{8}=k^3+\frac{3}{2}k^2+\frac{3}{4}k+\frac{3}{8}$ which is not whole number" is too vague so A0

:

Question	Scheme	Marks	AOs
16(i)	Explains that **a** and **b** lie in the same direction oe	B1	2.4
		(1)	
(ii)		M1	1.1b
	Attempts $\frac{\sin 30°}{6} = \frac{\sin\theta}{3}$	M1	3.1a
	$\theta =$ awrt $14.5°$	A1	1.1b
	Angle between vector $\mathbf{m}$ and vector $\mathbf{m}-\mathbf{n}$ is awrt $135.5°$	A1	3.2a
		(4)	
			(5 marks)

Notes

(i)

B1: Accept any valid response E.g The lines are collinear. Condone "They are parallel"
Mark positively. ISW after a correct answer
Do not accept "the length of line a +b is the same as the length of line a + the length of line b
Do not accept $|\mathbf{a}|$ and $|\mathbf{b}|$ are parallel.

(ii)

M1: A triangle showing 3, 6 and $30°$ in the correct positions.
Look for 6' opposite $30°$ with another side of 3.
Condone the triangle not being obtuse angled and not being to scale.
Do not condone negative lengths in the tringle. This would automatically be M0

M1: Correct sine rule statement with the sides and angles in the correct positions.
If a triangle is drawn then the angles and sides must be in the correct positions.
This is not dependent so allow recovery from negative lengths in the triangle.
If the candidate has not drawn a diagram then correct sine rule would be M1 M1
Do not accept calculations where the sides have a negative length. Eg $\frac{\sin 30°}{6} = \frac{\sin\theta}{-3}$ is M0

A1: $\theta =$ awrt $14.5°$

A1: CSO awrt $135.5°$

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